



CPM Model Applied to Turbulent Event Detections

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Abstract The present work shows the application of a change-point model (CPM), based on the Cramer von Mises test for turbulent structures detection in airflow. Specifically, it models the occurrence of vortices in the wake of an airfoil having a flow control device in its trailing edge. It seeks to compare the results obtained with conventional statistical methodologies employed in this type of analysis and the application of a change-point model. The main objective is to detect the characteristic frequencies of the turbulent structures immersed in the airflow.

The results show good response of the CPM methodology in the analysis of these flows, comparatively we observed the same values obtained by the methods mentioned above. This work shows a new application of change-point models for detecting changes in a time-dependent random signal which has an unknown distribution a priori.

Keywords change point, detection, turbulent scales, vortex, frequencies

1. Introduction

The problem of detecting a change has been a large area of research since the 50s. Because the problem has a very broad nature, the literature on the subject is very diverse and developed in many different fields. In particular, many of the methods have their origin in the community of quality control, where the main objective is to monitor the outputs of an industrial manufacturing process and detect faults in it as fast as possible [1].

However, there are many other applications where detection techniques are important, for example in disease monitoring, genetical data, intrusion into computer networks, evolution in financial markets, etc. For all of these issues there is much literature that refers to the subject, but its application to signal analysis, and in particular to data from measurements of fluctuating velocity components of airflow, is not well known.

Our goal then is to apply these familiar tools and some new ones which are being developed to our specific area of research. Mainly, we consider two primary objectives. One is to use these methodologies to incorporate hot wire anemometry as a tool that allows us to determine the occurrence of particular events in a fluid field in order to perform analysis and study. The other is its applicability as a tool in data acquisition software with the ultimate goal of optimizing the measurements performed using the scope of detection of any particular change in the signal.

In previous work [2], we reviewed the implementation of the change point methodology, from the concept of cumulative sum (CUSUM) [3, 4], used to detect small deviations in the mean or dispersion of the sample. In that work this methodology and its application to a specific case of measurements with hot-wire anemometry were detailed. The results were very encouraging, so the study and analysis was continued for other change point



models in order to determine which would be the most suitable to implement in our case.

2. Change point methodologies

As previously mentioned, this paper presents a change point model (CPM) obtained from recently published literature on the subject [5].

Currently, researchers have begun to work extensively on the subject and have defined certain basic criteria. Many statistical problems require the identification of change points in a data stream, as previously mentioned. Statistical process control (SPC) refers to the monitoring process that looks for a change in its distribution. Traditional methods assume that the distribution of the process is fully known prior to any change, including all its parameters. It is said that the process is "in control", if it can be generated by a prior distribution, and "out of control" if a change occurs that causes the process to be generated by different distributions. The final objective is to design SPC control charts that can monitor deviations from the distribution base. The most common way to evaluate the performance of control charts is the function of Average Run Length (ARL), where ARL_0 indicates the average of observations between the detection of false positives assuming that a change has occurred, and ARL_δ denotes the average delay before a change of size δ is detected. This is analogous to the classical idea of Type I bounded and Type II controlled error, applied in hypothesis test scenarios.

Historically, control charts were developed for the purpose of monitoring changes in the average value of a process, but today new procedures have been developed that allow standard deviation detections in both Gaussian as non- Gaussian distributions. This fact prompted us to investigate the applicability of these new methods to detect changes in a turbulent random signal.

The fact that the control charts traditionally require full knowledge of the process "in control" is not considered a problem when there is a large reference sample of observations that are known and used to generate a distribution "in control". This sample can be used to verify the assumptions made about the distribution and estimate any unknown parameters. This preliminary analysis of a sample of fixed size is called Phase I analysis, while the task of a sequential monitoring process when the observations are received in time is called Phase II analysis.

However, in some cases, the reference sample may be small or even not exist. In these cases, it would be impossible to accurately estimate the parameters "in control". This has important implications and many researchers have studied the impact of the lack of specification of parameters on the performance of control charts and found that even small deviations from the actual value may cause charts to show a significantly different compared to the desired ARL_0 value [6]. An even worse situation may occur when the distribution "in control" is incorrectly specified, as using a Gaussian distribution for processes that exhibit Skewness. Because of this situation, nonparametric control charts [7] that do not assume any knowledge of the distribution "in control" are needed (*free distribution charts*) and can maintain a desired ARL_0 value regardless of the true distribution of the process under study.

In this paper we include an alternative control chart that can detect arbitrary changes in the distribution of the process under study, for Phase I or Phase II monitoring, when you have very little knowledge about the shape of the distribution. A free distribution test is considered a CPM, and in this case we consider the Cramer-von-Mises test (CvM CPM), which is one of the most popular in nonparametric statistical literature [5].

First, we will introduce the concept of Phase I analysis. Consider the problem of detecting a change point in a fixed sequence of observations. Identifying the observations as $\{X_1, \dots, X_t\}$, the objective is to test if these have been generated by the same probability distribution. We assume that this distribution is not known a priori. Using the language of statistical hypothesis tests, the null hypothesis is that there are no concerns about change and all observations come from the same distribution, while the alternative hypothesis is that which indicates a change point τ that partitioned the sequence into two sets, with X_1, \dots, X_τ coming from the F_0 distribution before the change occurs and $X_{\tau+1}, \dots, X_t$ coming from a different distribution F_1 , after the change,

$$\begin{aligned} H_0 : X_i &\sim F_0 \quad \text{for } i = 1, \dots, t \\ H_1 : X_1, \dots, X_\tau &\sim F_0, \quad X_{\tau+1}, \dots, X_t \sim F_1 \end{aligned} \quad (1)$$

You can test the existence of a change point immediately after any observation, X_k , by partitioning the



observations into two samples $S_1 = \{X_1, \dots, X_k\}$ and $S_2 = \{X_{k+1}, \dots, X_t\}$ of sizes $n_1 = k$ and $n_2 = t - k$, respectively, and then applying a hypothesis test for two samples. We will use the CvM test for this, which is based on comparing the empirical distribution function of the two samples, as defined,

$$\hat{F}_{S_1}(x) = \frac{1}{k} \sum_{i=1}^k I(X_i \leq x) \quad (2)$$

$$\hat{F}_{S_2}(x) = \frac{1}{t-k} \sum_{i=k+1}^t I(X_i \leq x)$$

Where $I(X_i < x)$ is the indicator function

$$I(X_i < x) = \begin{cases} 1 & \text{si } X_i < x \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

This test uses a statistic based on the square of the average distance between the empirical distributions, and it can be estimated as

$$W_{k,t} = \sum_{i=1}^t |\hat{F}_{S_1}(X_i) - \hat{F}_{S_2}(X_i)|^2 \quad (4)$$

We reject the null hypothesis H_0 if $W_{k,t} > h_{k,t}$ for some threshold $h_{k,t}$

As it is not known where the change point will be located, we do not know which value of k to use for partitioning the sample. That is why we specify a more general hypothesis H_0 , i.e. there is no change in the sequence of values. The alternative hypothesis is then that there is a change point for some nonspecific value of k . Then we can make this test by calculating $W_{k,t}$ for each value $1 < k < t$ and take the maximum value. However, the statistical variance $W_{k,t}$ depends on the value of k . Because of this, we standardize the $W_{k,t}$ statistics so that they have equal mean and variance for all values of k . For our case, standardization is simple by using the known results of Anderson [8] and some basic algebra, thus the mean and variance of $W_{k,t}$ can be written as follows

$$\begin{aligned} \mu_{W_{k,t}} &= \frac{t+1}{6t} \\ \sigma_{W_{k,t}}^2 &= \frac{(t+1)[(1-3/4k)t^2 + (1-k)t - k]}{45t^2(t-k)} \end{aligned} \quad (5)$$

This leads to the maximization of the statistical test

$$W_t = \max_k \frac{W_{k,t} - \mu_{W_{k,t}}}{\sigma_{W_{k,t}}}, \quad 1 < k < t \quad (6)$$

If $W_t > h_t$ for a chosen suitably threshold h_t , then the hypothesis H_0 is rejected and we conclude that a change has occurred at some point in the data. In this case, the best estimator τ of the change point location is the k value that maximizes W_t . If $W_t \leq h_t$, then the hypothesis H_0 is not rejected and it is concluded that no change has occurred.

Having considered the problem of detecting change points from a fixed sample size, we will now have the task of monitoring the observations when they are being received one at a time (Phase II). Let X_t be the t^{th} observation, with t increasing with time. As a new observation X_t is received, we can treat $\{X_1, \dots, X_t\}$ like a fixed sample size and use the methodology described in the previous section to test if a change has occurred. Thus, the problem of sequential monitoring is reduced to develop a sequence of fixed-size test. Because of this, we analyzed the signal in Phase I condition to test the model.

One of the most important issues in the implementation of this CPM is the number of pre-change observations; this has a great impact on the performance of the model. As the prior distribution change is unknown, it will be easy to detect changes when the number of previous observations is large, making it possible to obtain a better estimated distribution and a more accurate empirical function distribution.



3. Applications of the CvM CPM and results

In analysis of turbulent flows it is common to directly apply well-known tools such as the spectrum power density, autocorrelations and wavelet transform to the values of velocity fluctuation. These values are obtained from tests performed in order to find the particular characteristics of the turbulent structures that appear in the flow. By employing these tools one can determine the occurrence of specific events present in the fluid field, times and spatial scales characteristics of these events [9]. In what follows, we will make a comparison of the results obtained by CPM methodology with those found with conventional tools.

We present the information from a series of experiments in which the application of the change point estimator CvM CPM shows its performance to detect changes in the signals acquired.

3.1. Injector case

We show measurements made with hot wire anemometry which sought to characterize the pulsating air flow jet from an injector nozzle designed for flow control experiments within a cavity of aspect ratio 1. Our interest, since this signal is not a free turbulent flow but a particular distribution of flow generated by the injector nozzle, was to see the behavior of the estimator being studied in order to establish its ability to detect changes in the signal, when the signal is fully understood.

At the output of the injector nozzle, in the vertical and transverse axis, a velocity field survey was undertaken to determine the injection velocity distribution. Moreover, the airflow was regulated with electrovalves (pulsating frequencies in an allowable range from 0 to 250 Hz.), in a pneumatic system (see Figure 1). Here we show two specific air pulse injection signals for the actuation frequency 10Hz and 100Hz, which are typical study cases. We applied Phase I monitoring to the data of the longitudinal velocity component in the axial extent of the air injector nozzle at a distance of 20 mm. The velocities were measured with a hot wire anemometer (constant temperature model Dantec Streamline TM.). A probe X-wire Dantec 55R51 with a data acquisition frequency of 4,000 Hz per channel was used, measuring two components of the velocity (longitudinal u and vertical v).

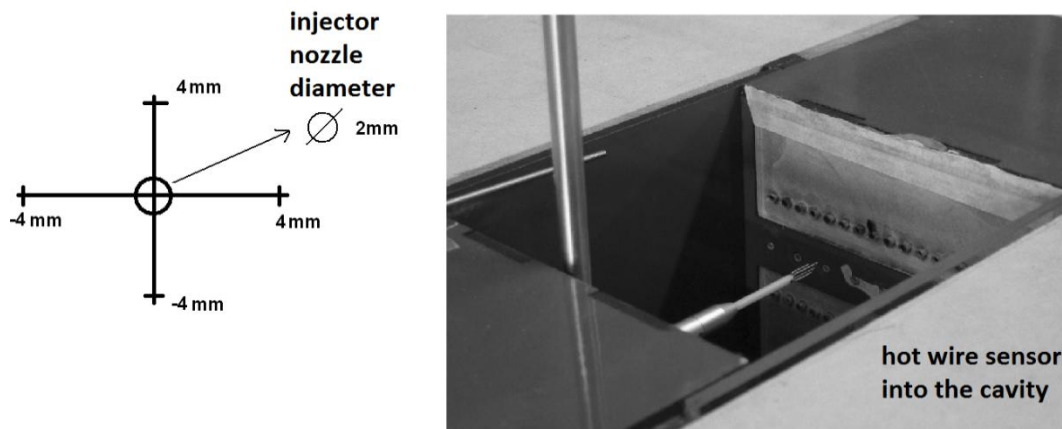


Figure 1: Jet velocity measurement scheme and cavity picture

Table 1: Injector velocity change point positions for both pulsation frequencies (partition of the signal)

Position	CvM CPM estimator	
	Time [s]	
	Pulsation 10Hz.	Pulsation 100Hz.
1	0.0450	0.0065
2	0.0960	0.0110
3	0.1455	0.0160
4	0.1955	0.0215
5	0.2435	0.0265
6	0.2945	0.0315
7	0.3435	-----



8	0.3940	-----
9	0.4430	-----
10	0.4935	-----
11	0.5425	-----
12	0.5930	-----
13	0.6420	-----
14	0.6925	-----
15	0.7415	-----

Table 1 shows the change points found in the signals of both injection frequencies, whereas a fraction of time of the total signal is observed. In Figures 2 and 3 we show fractions of the signals (for 10Hz. and 100Hz., changes may occur every 0.1 seconds and 0.01 seconds, respectively), indicating the position of the change points found by the estimator CvM (black vertical dotted lines). A good agreement with the initiation of the pulse and the drop in the signal at the end of the pulse is observed.

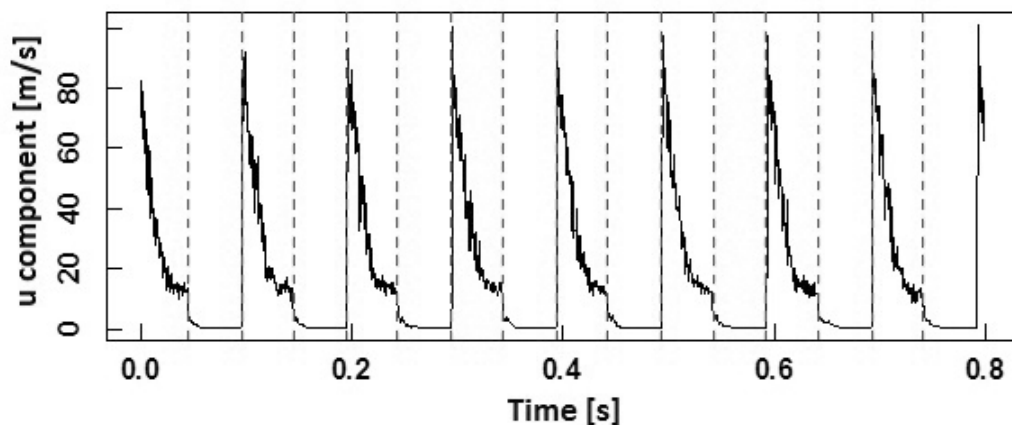


Figure 2: Injector axis u velocity component for pulsation frequency 10 Hz

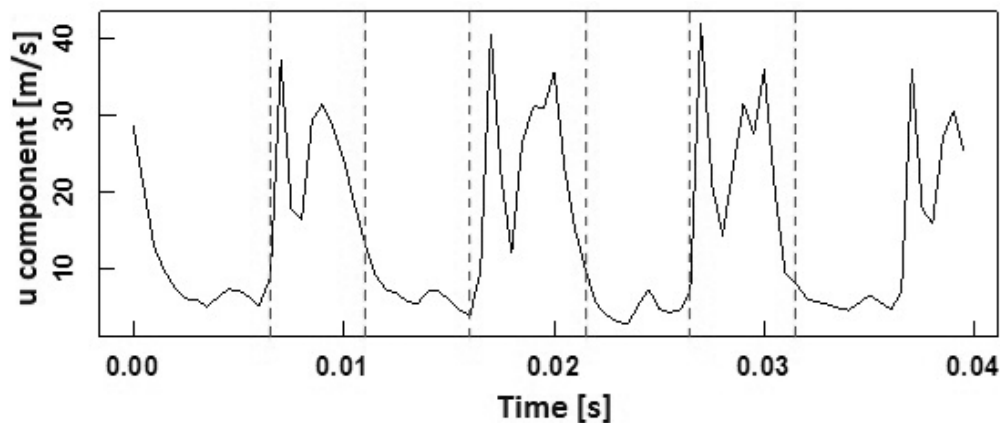


Figure 3: Injector axis u velocity component for pulsation frequency 100 Hz

3.2 Downstream signal from a flow control device

For the next case, the CUSUM methodology has already been previously applied in order to analyze the applicability of the CPM estimator to signal analysis [2], corresponding to a measurements taken in one of the boundary layer wind tunnels of our laboratory (UIDET-LaCLyFA), at the Aeronautic Department, Engineering College, at the National University of La Plata, having a test section 1m. high by 1.4m. wide. The model was a small wing with a 45 cm chord length (C) and 80 cm span (major wing length) built with a NACA 4412 airfoil in which a passive flow control device (Gurney mini-flap) was added with a length $h = 2\% C$, located on the trailing edge of the profile at 90° relative to the chord axis (see Figure 4). The measurement was performed at a mean flow velocity of 10 ms^{-1} , with a profile angle of attack of 0° (incidence chord angle relative to the free



stream direction), in order to get a Reynolds number of 300,000 for the test. These kinds of devices, incorporated to the profile at a 0° angle of attack, can increase the lift force by 60% and the maximum resistance value can be increased by 20% [10]. The disadvantage of this device is that the increase in lift is accompanied by an increase in drag, although the latter is not as significant. These effects are associated with vortex shedding in the wake of the mini flap.

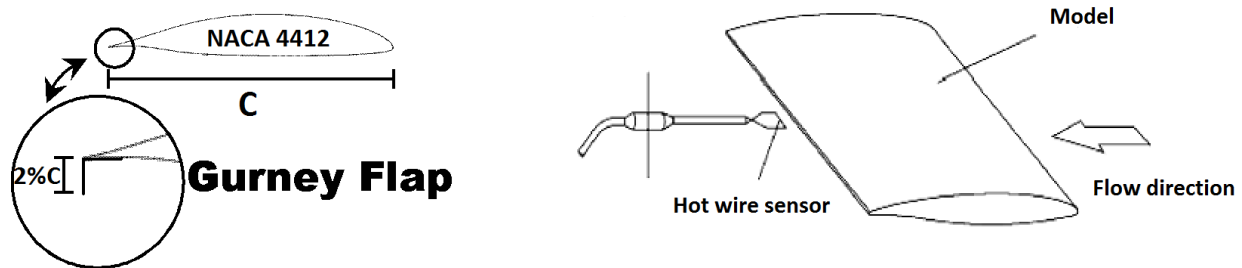


Figure 4: Model and measurement scheme

The velocities were measured with the previously mentioned equipment under the same conditions. The presented measurements correspond to a point in the wake generated by the profile at a distance of 1h downstream of the trailing edge, at the height of the chord, with the presence of the passive flow control device (Gurney mini-flap).

By knowing the flow field produced by the presence of this device and knowing that it generates periodic vortex structures, periodic counter rotating vortices (see Figure 5), the possibility of implementing this methodology to detect the expected wake events was considered. These events were identified by a power density spectrum, the calculation of the autocorrelation coefficients and applying the wavelet transform to the signal. The wavelets are localized in both space and frequency, therefore, the wavelet transform analyzes a signal locally in the frequency domain and space or time [11]. The characteristic frequency location in time of the wavelet transform gives a great opportunity to discover the positions of singularities and discontinuities in a signal, something that is impossible to achieve in the ordinary Fourier analysis [12]. These results with those standard methodologies and the change point models were compared.

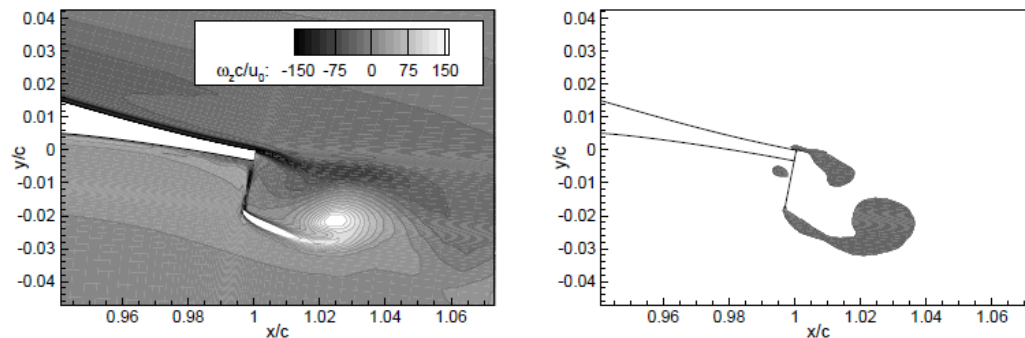


Figure 5: Counter rotating vortices scheme downstream the Gurney mini-flap and numeric simulations [10].

To compare, we present the result analysis found in the calculations for the vertical component of the velocity (v) of the analyzed signal. Figure 6 presents the wavelet map applying wavelet transform to the signal using a Mexican Hat wave type (*Ricker wavelet*), in which a maximum can be traced in the signal [12]. Hence, the occurrence of a turbulent periodic event associated with one of the counter-rotating vortices that are released downstream of the device is observed.

In Figure 6, the value corresponding to the scales ordinate is defined with the following expression:

$$Scale = \frac{\ln(\Delta t)}{\ln(10)} \quad (7)$$

Where Δt is the time in seconds of the length in time of the corresponding wave ("*Mexican hat*"), used for detecting the maximum.



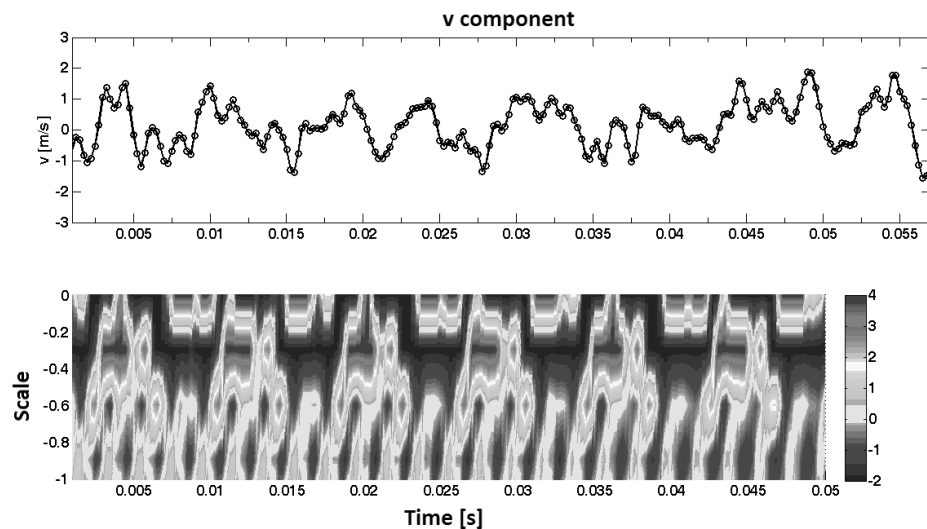


Figure 6: Wavelets map and time series of v component velocity fluctuations for the first 0.05 seconds (vortex scale: -0.3)

Structures marked on the map of wavelets (scale: -0.3) match the frequency of the peak power found by the analysis of the signal power density spectrum, as shown in Figure 7. There you can see a peak power at a frequency of 141.6 Hz. which corresponds to the frequency found in the corresponding secondary maxima at a time interval of 0.007 seconds.

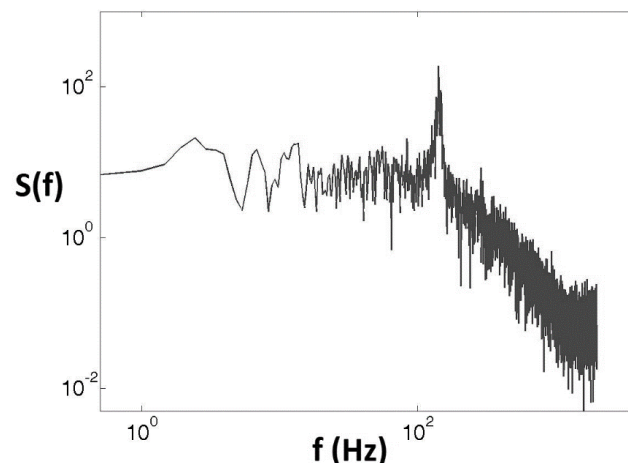


Figure 7: v component signal Power density spectrum

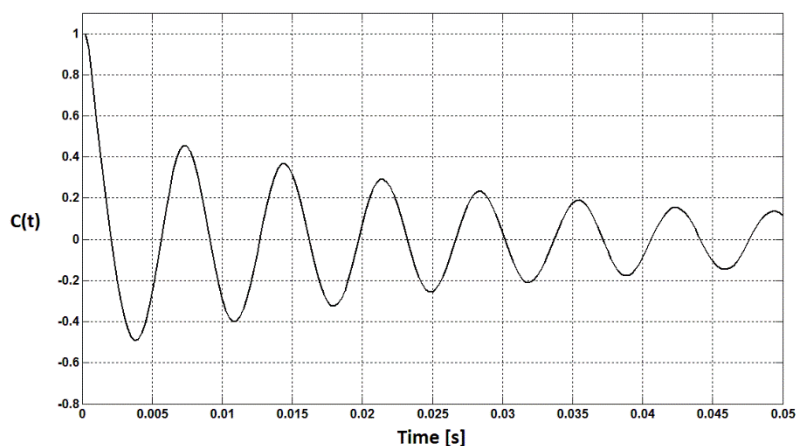


Figure 8: v component velocity fluctuation autocorrelation coefficient ($C(t)$)



To set the basic parameters in the detection of the known coherent structures, the autocorrelation of the fluctuations is added, as shown in Figure 8. In this Figure considering the theory of first cut by zero, the v component time scale of events are of 0.002 seconds, which can be translated by a given "frozen flow theory" multiplying the time scale (0.002 s) by the mean flow velocity (10 ms^{-1}), being the spatial scale of 0.02 m, which corresponds to a scale of magnitude of the device length. Therefore the detected structures correspond to the turbulent events (detected coherent turbulent structures) generated by the device.

Table 2: Signal change points results for both estimators CUSUM and CvM

Change points (Time [s])	
CUSUM Estimator	CvM Estimator
0.00250	0.00225
0.00475	0.00450
0.00900	0.00875
0.01225	0.01175
0.01575	0.01550
0.02000	0.02000
0.02200	0.02150
0.02475	0.02450
0.02675	0.02800
0.02925	0.03350
0.03375	0.03775
0.03775	0.04050
0.04075	0.04300
0.04400	0.04800

In Table 2 we show the change point results from the application of the change point estimator using the method of cumulative sums (CUSUM), against those found applying the CPM CvM methodology, at the same instant of time in seconds in which the change occurs in each case indicated. The results obtained show that the average time interval between changes is 0.0032 s.

Here we see that the change points found approximate maximum wavelet map for the scale of -0.3, with a frequency of 0.007 s, which correspond to known counter rotating vortex structures. Change point algorithm detects whether the maximum and minimum values observed in the wavelet map, which is a periodicity of about half the value obtained by other methodologies.

Figure 9 shows the fraction of the analyzed signal, again. Furthermore, in the same change points obtained by both methods are observed, indicating the same vertical lines with different color for each case. Thus, visually show the correlation of the results obtained. At some points it is not an exact correlation of the results, but overall both models show a good approximation in detecting signal changes, consistent with the effects on the wake for this particular flow.

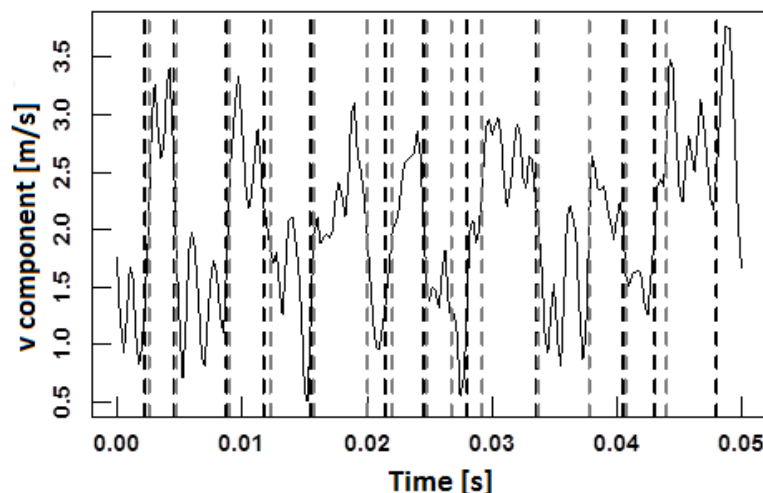


Figure 9: v component signal in the wake of the Gurney flap, (change points: CUSUM-black y CvM-grey)



4. Discussion and Conclusions

The CPM methodology proposed in this work shows its feasibility to be applied to experiments in the field of fluid turbulence, with two main purposes. First, both methods (CPM-CvM and CUSUM) show that they respond quite correctly for the detection of signal changes, which come from the events detection under study. Because of their satisfactory performance, the incorporation of these methods into the acquisition software would help to better optimize these types of tests. The calculations presented are based on the implementation of detection by monitoring Phase I. To which codes have been developed in R language, through libraries that have programed the change point models used (<http://cran.r-project.org/src/contrib/Archive/cpm/>).

The future implementation of the algorithms in the acquisition of data may be performed by calculating in suitable software for controlling the measurement, using the monitoring Phase II, whereby the analysis can be made of data sequentially. We seek, in this way, on-line detection of a possible change point and can optimize measurements using this methodology as a trigger (trigger) to sample start. Given the possible emergence of a coherent turbulent event, to be studied in detail, it will automatically begin data acquisition in order to analyze the characteristics of turbulence in the specific flow.

Finally, what is CPM advantage regarded others? As it is known, the most important turbulent analysis tool is the Wavelet transform, whose main advantage is obtaining the time and frequency domain of the events in the velocity signal, resulting in a valuable tool for turbulent flow analysis. Its main disadvantage is its prolonged calculation time, taking minutes to acquire results on a data acquisition pc. In contrast, a CPM takes fractions of a second to calculate. In this way, CPMs become a valuable tool for online turbulent flow event detection. Furthermore, CPM analysis generates the principal flow event which can be implemented as a trigger in the signal data acquisition, starting the measurement precisely when the event develops. On a broader scale, since wavelet transforms give all events detected in the signal, it will continue to be a useful tool for analysis. Thus, we consider that both the wavelet transform and CPM can complement each other as tools for turbulent flow analysis by utilizing the latter in real time and the former as a tool for comprehensive analysis of turbulent flow signal.

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